The Aircraft Boarding Problem

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Abstract

By minimizing boarding time, commercial airlines can improve their on-time performance and increase their aircraft/crew utilization and thereby increase profitability. Herein, we present preliminary results from combining IP and a simulation model that suggest that structured group boarding can result in boarding time reductions.

Keywords: Integer Programming, Nonlinear Assignment Problems, Airlines, Aircraft Boarding

1. Introduction

When boarding a commercial aircraft, the passengers are usually assigned to groups that determine the order that passengers board. The aircraft-boarding problem can be described as how to assign passengers to these boarding groups such that the total boarding time is minimized. A reduction in total boarding time can result in significant benefits for the airline industry. Boarding time is one of the significant elements of airplane turnaround time; i.e. the time between flights that an airplane spends on the ground. Airplanes produce revenues while flying; thus it is important that turnaround time be minimized. Turnaround time includes, airplane servicing, cargo handling, deplaning, and boarding. For many airlines, boarding is the bottleneck element in the process. Thus, reductions in boarding time result in revenue increases and/or cost reductions while potentially increasing passenger satisfaction.

The practice of most commercial airlines has been to board passengers in groups formed by assigning passengers sitting in contiguous rows to the same group. These groups are usually ordered from the back to the front of the aircraft (back-to-front approach), with special groups (usually first class and special needs passengers) boarding the aircraft before general boarding. The logic behind this boarding procedure is that the congestion of the aircraft aisle will be minimized by freeing the journey of the passengers to the back of the aircraft from aisle obstacles. However, it remains an open question whether this policy actually minimizes the total boarding time. An obvious problem with the back-to-front approach is that the congestion created in a reduced area of the aisle among passengers of the same group results in impediments for these passengers to stow their carry-on luggage and to reach their assigned seat in an expedient manner. This observation leads to the conjecture that a different boarding approach, where the groups are composed of passengers more dispersed throughout the aircraft, might actually perform better, than the current back-to-front approach. Previous researchers have already explored this conjecture using simulation.

In a study by Marelli, Mattocks, and Merry [2] different boarding strategies and different airplane interior configurations were tested on a Boeing 757 airplane using the Passenger Enplane/Deplane Simulation (PEDS) developed by the authors. This study showed that by boarding "outside-in," i.e. window-seats first, followed by middle seats and aisle seats last, boarding time could be decreased by as much as 17 minutes. The company "Shuttle by United" was one of the first companies to start using this outside-in strategy. While it was reported that the method was implemented with a good degree of success [3], the method was later discontinued and replaced by the current approach: first class first, followed by premier class, and the rest of the passengers boarding using the back-to-front approach.

Van Landeghem and Beuselinck [4] also used simulation to study numerous boarding strategies. Their study showed that the fastest boarding method consisted of passengers boarding individually according to their seat and row number. In addition, they also showed that by boarding by "half-row," i.e. splitting up of rows into two groups, significant boarding time reductions could be achieved.

Regarding the techniques used to analyze the aircraft-boarding problem, it seems that simulation has been the tool of choice. We are unaware of the use of any formal analytical, optimization model for this problem. Herein, we present an integer programming formulation of the aircraft-boarding problem. The problem is formulated as a nonlinear assignment problem where our objective is to minimize the total expected boarding time. We use the concept of "boarding interferences" as a surrogate measure of boarding time. We define a boarding interference as being an event where a passenger blocks the free flow of another passenger moving from the boarding gate to their seat. If one could board an aircraft without any passenger interference, improvements could be made only by changing other elements of the boarding process. Hence, we presume that there is equivalence between minimizing the total number of interferences and the total boarding time. In this paper, we present the development of the analytical model first, and then we build a simulation model of the aircraft-boarding procedure for an Airbus 320 (A320) airplane. We then perform a cross-validation between the analytical and simulation models. Finally, we use the simulation model to explore different aircraft-boarding scenarios.

2. Interference model

The minimization of total boarding time by assigning passengers to boarding groups is the goal of the aircraft-boarding problem. However, explicitly including time related parameters in an analytical model increases the complexity of the model representation and solution. Thus we resorted to a surrogate metric for time, the number of expected passenger interferences under a particular assignment strategy. We model the aircraft-boarding problem as a binary integer program with minimization of the total number of interferences as its objective function. We call this model the "interference model." We make the assumption, without a formal argument, that the minimization of interferences is equivalent to the minimization of total boarding time. In the interference model, we seek a grouping of passengers that will minimize the total number of expected interferences. An assumption of the model is that every passenger has been pre-assigned to a particular seat of the aircraft. A detailed discussion of interferences and the model development follows.

2.1 Types of interference

We define two types of interferences, *seat interference* and *aisle interference*. Seat interference occurs when a window or middle seat passenger boards later than the middle and/or aisle seat passenger that sits on the same side and same row of the aircraft. For example, assume a passenger is seated in seat 7C (aisle seat in row 7). When the passenger with seat 7B (middle seat in 7) boards the aircraft, passenger 7C must get out of their seat to allow passenger 7B access. There may be an even longer delay when passenger 7A (window seat in row 7) arrives and passengers 7B and 7C are already seated. Note that putting window-seat passengers in the groups boarding first would minimize this type of interference.

By aisle interference, we mean the situation when a passenger boarding the aircraft has to wait for the passenger in front of them to take their seat and to stow their luggage before proceeding to their seat, located further back in the aircraft. Aisle interferences involve two passengers; we refer to the "first" passenger as the passenger right ahead of the "second" passenger although they do not necessarily have to board as the first and second passengers in the boarding process. Aisle interference can occur within one group, or between two consecutive groups. Note that we assume passengers do not try to pass other passengers in the aisle of the aircraft.

2.2 Formulation of the aircraft-boarding problem

Consider an airplane having only one aisle with three seats on each side of the aisle (typical of Airbus 319 and 320, and the Boeing 737 and 757). Let $N = \{1, 2, 3, \ldots, n\}$ represent the set of rows and $M = \{A, B, C, D, E, F\}$ represent the set of seat positions in the aircraft. In addition, let the seats on the left side of the aisle be represented by $L = \{A, B, C\}$ and those on the right side by $R = \{D, E, F\}$, thus A and F are window seats, B and E are middle seats, and C and D are aisle seats. Given a row number $i \in N$ and a seat position $j \in M$, all seat locations in the aircraft can be uniquely identified and represented by the pair (i, j) just as in a normal aircraft, such as in seat $(7, C)$.

By assigning seats to groups (with a fixed seat assignment, this is similar to assigning passengers to groups) we are able to form groups of different sizes and composition. For the defined boarding problem, we want to assign each seat (i, j) to a boarding group k, with $k \in G$, with $G = \{1, 2, 3, \ldots, g\}$, where g is the total number of groups used in boarding the aircraft. Let the decision variable $x_{ijk} = 1$ if seat (i, j) is assigned to group k and $x_{ijk} = 0$ otherwise, for all $i \in N, j \in M, k \in G$.

The complete formulation of the aircraft-boarding problem is presented below. In this formulation, the equation numbers alongside the model serve to clarify the purpose of each set of expressions. In the objective function, we have different penalties for each type of interference. Seat interferences have penalties represented by l^s and aisle interferences by l^a. The penalties associated with the different types of interferences, capture their relative contribution to the total delay of the boarding procedure. These penalties are explained in detail in the next section.

Minimize: $$z = l_1^s \sum_{i \in N} \sum_{k \in G} x_{iAk} x_{iBk} x_{iCk} + l_1^s \sum_{i \in N} \sum_{k \in G} x_{iFk} x_{iEk} x_{iDk} + \qquad (1a)$$

$$l_2^s \sum_{i \in N} \sum_{k,l \in G: k<l} x_{iAk} x_{iBk} x_{iCl} + l_3^s \sum_{i \in N} \sum_{k,l \in G: k<l} x_{iAk} x_{iBl} x_{iCk} + l_4^s \sum_{i \in N} \sum_{k,l \in G: k<l} x_{iAl} x_{iBk} x_{iCk} + \qquad (1b)$$

$$l_2^s \sum_{i\in N} \sum_{k,l\in G:k<l} x_{iFk}\, x_{iEk}\, x_{iDl} + l_3^s \sum_{i\in N} \sum_{k,l\in G:k<l} x_{iFk}\, x_{iEl}\, x_{iDk} + l_4^s \sum_{i\in N} \sum_{k,l\in G:k<l} x_{iFl}\, x_{iEk}\, x_{iDk} +$$

$$l_5^s \sum_{i\in N} \sum_{k,l\in G:k<l} x_{iAk}\, x_{iBl}\, x_{iCl} + l_6^s \sum_{i\in N} \sum_{k,l\in G:k<l} x_{iAl}\, x_{iBk}\, x_{iCl} + l_7^s \sum_{i\in N} \sum_{k,l\in G:k<l} x_{iAl}\, x_{iBl}\, x_{iCk} + \tag{1c}$$

$$l_5^s \sum_{i\in N} \sum_{k,l\in G:k<l} x_{iDk}\, x_{iEl}\, x_{iFl} + l_6^s \sum_{i\in N} \sum_{k,l\in G:k<l} x_{iFl}\, x_{iEk}\, x_{iDl} + l_7^s \sum_{i\in N} \sum_{k,l\in G:k<l} x_{iFl}\, x_{iEl}\, x_{iDk} +$$

$$l_8^s \sum_{i\in N} \sum_{k,l,m\in G:k<l<m} x_{iAl}\, x_{iBm}\, x_{iCk} + l_9^s \sum_{i\in N} \sum_{k,l,m\in G:k<l<m} x_{iAk}\, x_{iBl}\, x_{iCm} + l_{10}^s \sum_{i\in N} \sum_{k,l,m\in G:k<l<m} x_{iAm}\, x_{iBl}\, x_{iCk} + \tag{1d}$$

$$l_{11}^s \sum_{i\in N} \sum_{k,l,m\in G:k<l<m} x_{iAk}\, x_{iBm}\, x_{iCl} + l_{12}^s \sum_{i\in N} \sum_{k,l,m\in G:k<l<m} x_{iAm}\, x_{iBk}\, x_{iCl} +$$

$$l_8^s \sum_{i\in N} \sum_{k,l,m\in G:k<l<m} x_{iFl}\, x_{iEm}\, x_{iDk} + l_9^s \sum_{i\in N} \sum_{k,l,m\in G:k<l<m} x_{iFk}\, x_{iEl}\, x_{iDm} + l_{10}^s \sum_{i\in N} \sum_{k,l,m\in G:k<l<m} x_{iFm}\, x_{iEl}\, x_{iDk} +$$

$$l_{11}^s \sum_{i\in N} \sum_{k,l,m\in G:k<l<m} x_{iFk}\, x_{iEm}\, x_{iDl} + l_{12}^s \sum_{i\in N} \sum_{k,l,m\in G:k<l<m} x_{iFm}\, x_{iEk}\, x_{iDl} +$$

$$l_1^a \sum_{i\in N} \sum_{u,v\in L:v\neq u} \sum_{k\in G} x_{iuk}\, x_{ivk} + l_1^a \sum_{i\in N} \sum_{u,v\in R:u\neq v} \sum_{k\in G} x_{iuk}\, x_{ivk} + \tag{2a}$$

$$2\, l_2^a \sum_{i\in N} \sum_{u,v\in M:u\in L,v\in R} \sum_{k\in G} x_{iuk}\, x_{ivk} + \tag{2b}$$

$$l_3^a \sum_{a,b\in N:a<b} \sum_{u,v\in M} \sum_{k\in G} x_{auk}\, x_{bvk} + \tag{2c}$$

$$l_4^a \sum_{i\in N} \sum_{u,v\in M} \sum_{k,l\in G:k<l} x_{iuk}\, x_{ivl} + l_4^a \sum_{i\in N} \sum_{u,v\in M} \sum_{k,l\in G:k<l} x_{iuk}\, x_{ivl} + \tag{2d}$$

$$l_5^a \sum_{i\in N} \sum_{u,v\in M:u\in L,v\in R} \sum_{k,l\in G:k<l} x_{iuk}\, x_{ivl} + l_5^a \sum_{i\in N} \sum_{u,v\in M:u\in L,v\in R} \sum_{k,l\in G:k<l} x_{ivl}\, x_{iuk} + \tag{2e}$$

$$l_6^a \sum_{a,b\in N:a<b} \sum_{u,v\in M} \sum_{k,l\in G:k<l} x_{auk}\, x_{bvl} \tag{2f}$$

Subject to:

$$\sum_{k\in G} x_{ijk} = 1 \qquad \text{for all } i\in N,\ j\in M \tag{3}$$

$$\sum_{i\in N} \sum_{j\in M} x_{ijk} \geq C_{\min} \qquad \text{for all } k\in G \tag{4}$$

$$\sum_{i\in N} \sum_{j\in M} x_{ijk} \leq C_{\max} \qquad \text{for all } k\in G \tag{5}$$

$$x_{ijk} \in \{0,1\} \qquad \text{for all } i\in N,\ j\in M\ ,\ k\in G \tag{6}$$

Expressions (1a) through (1d) are associated with seat interferences. Seat interferences may occur when: all seats of the same side in one row are assigned to the same boarding group (1a); when two seats of the same side and row are assigned to one boarding group and the third seat is assigned to a later (1b) or earlier (1c) boarding group; or when all seats of the same side and row are assigned to different boarding groups (1d).

The aisle interferences are represented by the expressions numbered from (2a) to (2f). (2a), (2b), and (2c) represent the aisle interferences that take place within a group; (2d), 2(e), and (2f) are the aisle interferences that take place between two consecutive groups. Aisle interferences can occur when the "first" and "second" passenger are seated in the same row and same side (2a) and (2d), same row and different side (2b) and (2e), and when the "second" passenger has a higher row number than the "first" passenger (2c) and (2f) (in this case, side does not matter). Note that we could have taken the expressions (2a, b, c), similarly (2d, e, f), into one expression. However, this would not allow us to apply different penalties to these seemingly different interferences.

The constraints grouped under (3) represent the assignment restrictions. These constraints ensure that each seat is assigned to only one boarding group. The constraints grouped under (4) and (5) restrict the group size to at least $C_{\min}$ and at most $C_{\max}$ seats assigned to each group. Finally, (6) are binary constraints.

2.3 Determination of penalties

Values for the penalties can be determined using a variety of procedures; e.g., using historical data one could estimate the contributions of each type of interference to the total delay. The procedure that is followed in this paper is based on probabilistic expectations. The main assumption in the determination of these expectations is that the boarding position of a particular passenger within the group is equally likely. That is, a particular passenger within a group can take any of different boarding positions within that group, with the same probability. Based on these assumptions, the expected number of interferences will be computed and used as the penalty represented in the objective function of the model by the λ's.

As an example consider the following: Suppose that three passengers on the same side of a row are assigned to the same boarding group, passengers sitting in positions *A* (window), *B* (middle), and *C* (aisle). For purposes of computing the seat interferences penalty, there are six different ways for these passengers to board the plane (*ABC*, *ACB*, *BAC*, *BCA*, *CAB*, *CBA*). Based on our assumptions, each of these boarding patterns is equally likely; however, the interferences caused by each differ. For example, the boarding pattern *ABC* implies that the window passenger (*A*) will board the plane first, passenger sitting in the middle seat (*B*) boards next (not necessarily immediately), and finally the passenger sitting in the aisle seat (*C*). This represents the best-case scenario (zero penalty) because no passenger would have to get up once seated. Now consider the boarding pattern *ACB* where the window seat passenger boards before the aisle seat, who boards before the middle seat. In this case, there is one interference, the aisle seat passenger getting up for the middle seat passenger. Since each one of the boarding patterns is equally likely (1/6 probability), once we determine the interferences associated with each one of the patterns it is straightforward to obtain the expected value. The same procedure is used when the different passengers of the same row and aisle side are assigned to different groups. For instance, suppose that the window and aisle passengers are assigned to the same boarding group. We will have one interference if the aisle passenger boards before the window passenger and none if the window seat passenger boards first. Since the probability of each of the two boarding patterns is 0.5 and since we know that the middle passenger will be assigned to a different group causing an interference of one with probability of one, the total expected interferences, and the penalty, for this case is 1.5.

In Table 1, all expected interferences greater than zero with the corresponding penalty l^s are shown. The squared brackets '[ ]' imply passengers boarding in the same group and the arrow shows the order of the boarding when the passengers board in different groups. The order of the passengers within the boarding groups is assumed to be random.

Table 1. Expected seat interference

Penalty	**Passenger order**	**E(#Interferences)**
l_1^s	[window, middle, aisle]	1.5
l_2^s	[window, middle] $\rightarrow$ [aisle]	0.5
l_3^s	[window, aisle] $\rightarrow$ [middle]	1.5
l_4^s	[middle, aisle] $\rightarrow$ [window]	2.5
l_5^s	[window] $\rightarrow$ [middle, aisle]	0.5
l_6^s	[middle] $\rightarrow$ [window, aisle]	1.5
l_7^s	[aisle] $\rightarrow$ [window, middle]	2.5
l_8^s	[window] $\rightarrow$ [aisle] $\rightarrow$ [middle]	1
l_9^s	[middle] $\rightarrow$ [window] $\rightarrow$ [aisle]	1
l_{10}^s	[middle] $\rightarrow$ [aisle] $\rightarrow$ [window]	2
l_{11}^s	[aisle] $\rightarrow$ [window] $\rightarrow$ [middle]	2
l_{12}^s	[aisle] $\rightarrow$ [middle] $\rightarrow$ [window]	3

In a similar way, we can calculate the expected aisle interferences. For aisle interferences within a boarding group, we are looking for the number of ways that two passengers can interfere with each other. Consider a boarding group of size s_1. One type of aisle interference is when the "second" passenger (the passenger boarding last) has a higher row number (further back the plane) than the "first" passenger. If we assume the size of their boarding group to be equal to s_1, then there are $(s_1 - 1)(s_1 - 2)!$ out of $s_1!$ ways the passengers could have boarded the airplane such that the "first" and "second" passenger interfere with each other. There are (s_1 – 1) positions for the two passengers to board after one another leaving (s_1 – 2)! ways for the other passengers to board in a group of size s. With s! total ways to board the s passengers, the probability that the "first" and "second" passenger interfere is equal to 1/ s_1 (= (s_1 – 1)(s_1 – 2)!/ s_1!). If there are m passengers with a lower row number than the "second" passenger, the expected value for this type of aisle interferences is m/s_1.

For aisle interferences between groups, the probability that the "first" passenger will be last in his group is $1/s_1$ (assume a group size of s_1), and the probability that the "second" passenger will be first in the succeeding group is $1/s_2$ (assume a group size of s_2). Hence, the probability of interference is $1/(s_1s_2)$. Hence, with m possibilities the expected value for the between group aisle interferences is equal to $m/(s_1s_2)$. Table 2 summarizes the penalties.

Table 2. Expected aisle interference

Penalty	**Description**	**E(#Interferences)**
$l_1^a,\ l_2^a,\ l_3^a$	Within groups	$1/s_1$
$l_4^a,\ l_5^a,\ l_6^a$	Between groups	$1/(s_1s_2)$

3. Computational results

The interference model is a nonlinear assignment problem with quadratic and cubic terms in the objective function. Assignment problems like these belong to the NP-hard complexity class. We used the NEOS Server for MINLP and resort to the heuristics implemented in this solver. MINLP solves mixed integer nonlinearly constrained optimization problems by using a branch-and-bound algorithm where the nodes correspond to continuous nonlinearly constrained optimization problems. Detailed MINLP information can be found in the user manual [1].

The model was solved using an A320 aircraft configuration. Most A320 have twenty-six rows of which the first three are first class.

Figure 1 shows eight different strategies using different numbers of groups. Each figure shows a layout of an A320 aircraft. The number shown on each seat is the boarding group to which that seat was assigned under the different strategies. The first four strategies shown represent the "traditional" back-to-front approach and the next four are found by MINLP. Since these last four have a tendency to board outside first, we refer to these as outside-in.

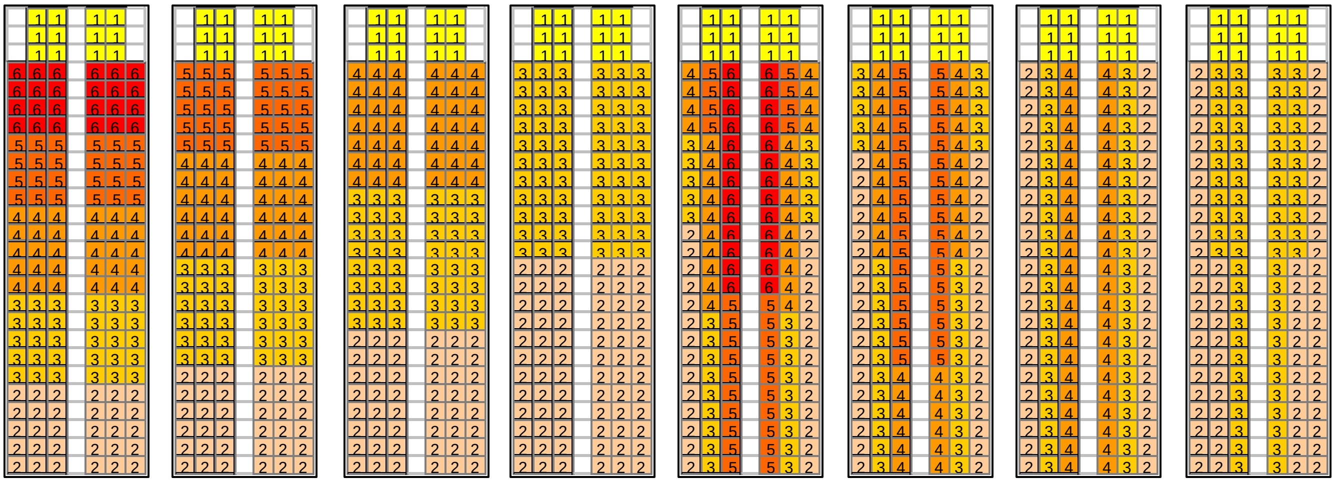
Table 3 summarizes the number of expected interferences for each of the strategies in Figure 1. It can be observed that outside-in boarding outperforms the back-to-front approach. The main reduction comes from the seat interferences, but even on the total expected number of aisle interferences outside-in, reverse-pyramid like, performs better. It remains a question, however, how important seat and aisle interferences are with respect to total time.

4. Simulation

A simulation model was implemented to provide some level of validation of the analytical model and to provide a finer level of detail. Data on time between passengers, walking speed, interference time, and time to store luggage in the overhead bins were recorded from videotaping actual aircraft boarding procedures. Two cameras were used, one inside the aircraft and one inside the jet-bridge. The simulation model was built in ProModel 2001 and collects statistics on seat and aisle interferences and total boarding time. Each boarding strategy was run 100 times and the results are shown in Table 4. This number of replicates was large enough for all tested strategies to give 95% confidence intervals of less than sixty seconds. We can clearly see that the strategies based on the solutions of the interference model are better than the back-to-front approach. The average number of seat interferences match well with the number calculated in the interference model. On the other hand, the number of aisle interferences in the simulation is significantly lower than in the interference model. The reason for this difference is that the way that aisle interferences are defined in the analytical model does not always results in an aisle interference in the simulation model. However, the trend of aisle interferences in the analytical model corresponds almost exactly to the trend observed in the simulation model (correlation coefficient of over 0.80).

5. Conclusions

This paper discusses an integer programming approach for the aircraft-boarding problem. The approach assigns seats (passengers) to boarding groups such that expected interferences are minimized. The interference model shows that outside-in boarding outperforms traditional back-to-front boarding. Although the model makes several simplifying assumptions, it provides a good insight on the boarding process. In addition, simulation shows that the outside-in strategy works better than back-to-front; however, the latter is most commonly used in practice. More analysis is needed to demonstrate the effectiveness of the outside-in strategy. The strategy is currently being tested on a pilot basis by a major commercial airline in the United States.



BF6 BF5 BF4 BF3 OI6 OI5 OI4 OI3

Figure 1. Back-to-front (BF) and outside-in (OI) boarding strategies showing the seat assignment to boarding groups

Table 3. Number of expected interference by boarding strategy

	BF6	BF5	BF4	BF3	OI6	OI5	OI4	OI3
Seat Interferences	**72**	**72**	**72**	**72**	**3**	**3**	**3**	**26**
First class [xx]	3	3	3	3	3	3	3	3
First class [x] [x]	0	0	0	0	0	0	0	0
Coach class [xxx]	69	69	69	69	0	0	0	0
Coach class [xx] [x]	0	0	0	0	0	0	0	12
Coach class [x] [xx]	0	0	0	0	0	0	0	11
Coach class [x] [x] [x]	0	0	0	0	0	0	0	0
Aisle Interferences	**87**	**85**	**83**	**81**	**78.681**	**78.404**	**78.043**	**78.686**
Within one group								
Same row same side	11	9	7	5	1	1	1	2.333
Same row different side	17	14	11	8	7	6	5	5.333
Different rows	58	61	64	67	68	69	70	69.667
Between groups								
Same row same side	0	0	0	0	0.059	0.058	0.043	0.019
Same row different side	0	0	0	0	0.059	0.058	0.043	0.019
Different rows	1	1	1	1	2.562	2.288	1.957	1.314
Total Interferences	**159**	**157**	**155**	**153**	**81.681**	**81.404**	**81.043**	**104.686**

Table 4. Number of interference by boarding strategy, 100 runs simulation results

	BF6	BF5	BF4	BF3	OI6	OI5	OI4	OI3
Avg. Seat Interferences	72.22	73.36	72.11	70.76	2.94	2.94	2.94	26.05
Avg. Aisle Interferences	52.27	52.74	53.36	53.41	42.64	42.92	42.02	46.95
Avg. Total Interferences	124.49	126.1	125.47	124.17	45.58	45.86	44.96	73
Avg. Boarding Time (sec)	1491.68	1473.69	1460.68	1436.76	1387.8	1382.71	1376.07	1412.79

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